PG-CS-1124 **MMSS–31**

P.G. DEGREE EXAMINATION - FEBRUARY, 2023

Mathematics

Third Semester

TOPOLOGY

Time : 3 hours Maximum marks : 70

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight Questions in 300 words.

All questions carry equal marks.

1. If A is a subspace of X and B is a subspace of Y, then show that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$.

2. If A is a subset of the topological space X, then prove that $x \in \overline{A}$ if and only if every open set U containing x intersect A.

3. Show that the union of a collection of connected subspaces of X that have a point in common is connected.

4. Show that every compact Hausdorff space is normal.

5. Define a completely regular space. Show that a subspace of a completely regular space is completely regular.

6. State and prove uniform limit theorem.

7. If $f : X \to Y$ is a continuous map of the compact metric space (X, d_X) to the metric space (Y, d_Y), prove that f is uniformly continuous.

8. Show that a subspace of a regular space is regular.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions out of Five questions in 1000 words.

All questions carry equal marks.

9. (a) Let (X, τ) be a topological space and $\mathcal{C}$ is a collection of open sets of K such that for each open set U of X and each $x \in U$ there exists $C \in \mathcal{C}$ such that $x \in C \subset U$. Show that $\mathcal{C}$ is a basis for the topology τ on X. (12)

(b) Define a topology and give an example. (3)

10. Let X and Y be topological spaces. Let $f : X \to Y$. Then show that the following are equivalent.

    (a) f is continuous

    (b) For every subset A of X, one has $f(\overline{A}) \subset \overline{F(A)}$.

    (c) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

    (d) For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

11. If L is a linear continuum in the order topology, then show that L is connected and are rays in L.

12. Show that every regular space with a countable basis is normal.

13. State and prove the Tietze extension theorem.

PG-CS-1125	MMSS–32

P.G. DEGREE EXAMINATION – FEBRUARY, 2023.

Mathematics

Third Semester

FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight Questions in 300 words.

All questions carry equal marks.

1. Define a Banach space and show that $\mathbb{R}^n$ where $\mathbb{R}$ is a set of real numbers, is a Banach space.

2. If N is a normed linear space and x_0 is a non-zero vector in N, then show that there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.

3. Let S be a non-empty subset of a Hilbert space H, show that

   (a) $S \subset S^{\perp\perp}$ and

   (b) $S \cap S^{\perp} = \{0\}$.

4. Show that the adjoint operation $T \to T^*$ on $\mathcal{B}$(H) has the following properties.

   (a) $(T_1 T_2)^* = T_2^* T_1^*$

   (b) $\|T^*T\| = \|T\|^2$.

5. Show that the spectrum of an element x, $r(x)$ is non-empty.

6. State and prove the uniform boundedness theorem.

7. State and prove the parallelogram law.

8. Show that an operator T on a Hilbert space H is unitary if and only if it is isometric isomorphism of H onto itself.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 1000 words.

All questions carry equal marks.

9. If N is a normed linear space and N' is a Banach space, then show that the set $\mathcal{B}(N, N')$ the set of all continuous linear transformations of N into N', is a Banach space.

10. State and prove the open mapping theorem.

11. Let H be a Hilbert space, and let f be an arbitrary functional in H^*. Show that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H.

12. (a) If T is an operator on H for which $(Tx, x) = 0$ for all x, then show that $T = 0$.

    (b) Show that if $P_1, P_2, ..., P_n$ are projections on closed linear subspaces $M_1, M_2, ..., M_n$ of H, then show that $P = P_1 + P_2 + ... + P_n$ is a projection if and only if the P_i's are pairwise orthogonal and in this case, P is a projection on $M = M_1 + M_2 + ... + M_n$. (7 + 8)

13. Show that the spectral radius $r(x) = \lim \|x^n\|^{\frac{1}{n}}$.

PG-CS-1126 **MMSS–33**

P.G. DEGREE EXAMINATION - FEBRUARY, 2023

Mathematics

Third Semester

ORDINARY DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 70

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions out of Eight Questions in 300 words.

All questions carry equal marks.

1. Solve the initial value problem $4y''-8y'+3y=0$, $y(0)=2, y'(0)=\frac{1}{2}$.

2. Suppose $\phi_1, \phi_2, \ldots \phi_n$, be n solution of $L(y)=0$ on an interval I containing a point x_0 then show that $W(\phi_1 \ldots . \phi_n)(x)=e^{-\alpha_1(x-x_0)}W(\phi_1 \ldots . \phi_n)(x_0)$.

3. Show that $\int_{-1}^{1} P_n(x)P_m(x)\,dx=0$. where $n \neq m$.

4. State and prove the existence theorem for analytic coefficients.

5. Show that –1 and 1 are regular singular point for the Legendre equation

$(1-x^2)y''-2xy'+\alpha(\alpha+1)y=0$

6. Compute the indicial polynomial for the equation $x^2y''+a(x)xy'+b(x)y=0$ where a, b have convergent power series, expansions for $|x|<r_0$, $r_0>0$.

7. Write a brief notes on Lipschitz condition.

8. The successive approximations, ϕ_k defined by $\phi_0(x)=y_0$, exist as continuous functions on $I:|x-x_0|\leqq = a$ = minimum $\left\{a,\frac{b}{m}\right\}$, and $(x,\phi_k(x))$ is in R for x in I, then show that the ϕ_k satisfy $|\phi_k(x)-y_0|\leqq M|x-x_0|$ for all x in I.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions out of Five questions in 1000 words.

All questions carry equal marks.

9. State and prove the existence and uniqueness theorem of IVP.

10. Suppose ϕ be any solution of $L(y)=0$ on an interval I contain a point x_0 then for all x in I and show that $\|\phi(x_0)\|e^{-k|x-x_0|}\leqq\|\phi(x_0)\|\leqq\|\phi(x_0)\|e^{k|x-x_{0.}|}$, where $k=1+|a_1|+...+|a_n|$.

11. (a) Show that $\int_{-1}^{1} P_n^2(x)\,dx = \frac{2}{2n+1}$. (10)

(b) Show that there exist n linearly independent solution of $L(y)=0$ on 1. (5)

12. Compute the solution for the bassel equation $x^2 y'' + xy' + (x^2 - a^2)y = 0$ of order a where α is constant, Re $\alpha \geqq 0$.

13. Suppose M, N be two real valued functions which have continuous first partial derivatives on some rectangle $R: |x - x_0| \leqq a, |y - y_0| \leqq b$, then prove that the equation $M(x,y) + N(x,y)y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

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PG–CS-1127 **MMSS–34**

P.G. DEGREE EXAMINATION — FEBRUARY 2023.

Mathematics

Third Semester

NUMERICAL ANALYSIS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight questions in 300 words.

All questions carry equal marks.

1. Solve the system of equation using gauss elimination method $3x + y - z = 3$, $2x - 8y + 2 = -5$, $x - 2y + 9z = 8$.

2. Find the dominant of eigen value and eigen vector of $A = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$ using power method.

3. Using Lagrange's formula, calculate $f(3)$ from the following table

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

4. Find the value of $\log 2^{\frac{1}{3}}$ from $\int_0^1 \frac{x^2}{1+x^3} dx$ using Simpson's one third rule with $h = 0.25$.

5. Write a brief note on Chebyshev polynomials

6. For each appropriate function $f(x)$ there is a unique least squares polynomial approximation of degree at most n which minimizes.

7. Using Taylor series method, compute the value of $y(0.2)$ correct to 3 decimal places from $\frac{dy}{dx} = 1 - 2xy$ given that $y(0) = 0$.

8. Find the solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to $u(x,0) = \sin \pi x, 0 \le x \le 1,\quad u(x,0) = u(1,t) = 0$ using Schmidt method.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE questions out of Five questions in 1000 words.

All questions carry equal marks.

9. Solve the following system of equation by Gauss Seidal iteration method

$10x_1 - 2x_2 - x_3 - x_4 = 3$, $-2x_1 + 10x_2 - x_3 - x_4 = 15$,
$-x_1 - x_2 + 10x_3 - 2x_4 = 27$,
$-x_1 - x_2 - 2x_3 + 10x_4 = -9$

10. (a) The population of a town in the census is as given in the data estimate the population in the year 1996 using Newton's backward interpolation formula (8)

Year(x)	1961	1971	1981	1991	2001
Population(y) (in 1000s)	46	66	81	93	101

(b) From the following table, find the value of tan 45° 15' by using Newton forward interpolation method. (7)

x^{o}	45	46	47	48	49	50
$\tan x^{o}$	1.00000	1.03553	1.07237	1.11061	1.15037	1.19175

11. Suppose $f(x)$ be continuous on [a, b] and $Q_n(x) = \sum_{j=0}^{n} C_j P_j(x)$, where n=0, 1, 2, ..., and $c_j = \int_a^b P_j(x) f(x) dx, \quad j = 1, ..., n$ be the least squares polynomial approximations to $f(x)$ on [a, b]. The prove that $\lim_{n\to\infty} J_n = \lim_{n\to\infty} \int_a^b [f(x) - Q_n(x)]^2 dx = 0$ and have Parseval's equality $\int_a^b f^2(x) dx = \sum_{j=0}^{\infty} c_j^2$.

12. Find the values of $y(0.2)$ and $y(0.4)$ using Runge-kutta method of fourth order with $y(0) = 1$ given that $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$.

13. Given that $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $u(0,t) = 0$, $u(4,t) = 0$ and $u(x,0) = \frac{x}{3}(16 - x^2)$. Find u_{ij} where $i = 1,2, 3, 4$ and j=1, 2 by using Crank nicholson's method.

PG-CS-1128	MMSSE-5

P.G. DEGREE EXAMINATION – FEBRUARY 2023.

Mathematics

Third Semester

GRAPH THEORY

Time : 3 hours Maximum marks : 70

PART A — $(5 \times 5 = 25$ marks)

Answer any FIVE questions out of Eight questions.

All questions carry equal marks.

1. Define the following of a graph G:

   (a) Radius

   (b) Diameter

   (c) Girth.

2. Prove that there are exactly two isomorphism classes 4-regular simple graphs with 7 vertices.

3. Prove that if G is Hamiltonian then for every non-empty proper subset S of V, $\omega(G - S) \leq |S|$.

4. Apply Mycielskian construction for 5-cycle graph and sketch the resulting graph.

5. Prove that K_5 is non-planar.

6. Find the chromatic polynomial of K_4.

7. Briefly write about use of Wang and Kleitman's algorithm.

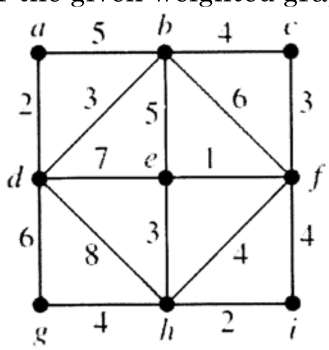
8. Define the following
   (a) Matching
   (b) Perfect Matching
   (c) Maximum matching in a graph G.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions out of Five questions

All questions carry equal marks.

9. Use Prim's algorithm to find a minimum spanning tree (MST) for the given weighted graph.



10. State and Prove Menger's theorem.

11. Prove that a simple graph G is Eulerian if and only if it is connected and every vertex has an even degree.

12. State and prove Vizing's theorem.

13. State and prove Euler's formula on planarity.
